

Tool A: Sample Size Calculation for a Specific Scenario
Planning that Takes into Account FPC, Cluster Effects and Prior Knowledge
(Bayesian)

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1 Methodological Setup and Context

This planning report calculates the minimum sample size required to estimate the prevalence of a binomially distributed trait in a herd with a predefined precision (Cochran, 1977; Dohoo et al., 2003; Thrusfield et al., 2018). It is part of a framework for sample size planning of binomially distributed traits (Kulig et al., 2026).

Focus scenario:

- **Herd size (N):** 120 animals (finite population correction applied)
- **Assumed prevalence (π):** 50 %
- **Required precision (margin of error, MoE):** ± 5 percentage points (pp) (*The planned interval therefore ranges from about 45 % to 55 %.*)
- **Targeted confidence level:** 95 % (*Interpretation: The MoE defines the **width** of the interval, the confidence level the **reliability** of the procedure. If the sampling were repeated 100 times in exactly the same way, about 95 of the resulting confidence intervals would contain the true herd prevalence. For the Bayesian credible interval, the level is instead interpreted as the posterior probability that the true prevalence lies within the interval (Gelman et al., 2013).*)

Bayesian prior knowledge:

- **Selected prior:** Beta(1, 1)
- **Interpretation:** A flat, non-informative prior is used. The model assumes that no prior knowledge about the prevalence exists.

Cluster structure:

- **Cluster sampling active:** No (simple random sampling)
- **Pens actually available:** Not defined (assumed unlimited)
- **Resulting design effect (D_{eff}):** 1

2 Recommendation of the Statistical Method

The choice of an appropriate interval method depends primarily on the expected prevalence and on the ratio of the sample size to the population size (Brown et al., 2001; Newcombe, 1998).

2.1 Assessment of the Wald Approximation

The classical normal approximation (Wald method) assumes a symmetric sampling distribution of the estimated proportion. The approach goes back to Laplace (1812) and is named after the Wald test (Wald, 1943), from whose inversion the interval results (Agresti & Coull, 1998). **Status:** The assumed prevalence (50 %) lies in the central range (20 % to 80 %).

Here the Wald method yields stable and acceptable sample sizes. Because its actual coverage fluctuates even for moderate proportions, the Wilson score interval nevertheless remains preferable when maximum methodological rigour is required (Agresti, 2007; Brown et al., 2001).

2.2 Finite Population Correction (FPC)

Animals are sampled without replacement. The larger the proportion of the herd that is examined, the more information is available about the *actual* remainder of the herd, and the smaller the uncertainty becomes. This is taken into account by the finite population correction (Cochran, 1977; Kish, 1965):

$$n_{fin} = \frac{n_{\infty}}{1 + (n_{\infty} - 1)/N}$$

As a rule of thumb, the FPC should be applied when the calculated sample size exceeds 5 % of the herd (Cochran, 1977) (here $N = 120$).

Status: The uncorrected sample size would amount to about 320.1 % of the herd. **The FPC is therefore essential** and yields a considerably smaller net sample size, because a substantial part of the herd is examined.

3 Scenario Matrix: Comparison of Sample Sizes

The following table shows the calculated net sample size (n_{fin}) for different confidence and precision requirements. The values include the finite population correction (FPC, if activated) but not yet the design effect. The four methods are the Wald/Cochran approximation (Cochran, 1977), the Wilson score interval (Wilson, 1927), the exact Clopper-Pearson interval (Clopper & Pearson, 1934) and the Bayesian equal-tailed credible interval based on the conjugate Beta-binomial model (Gelman et al., 2013; Joseph et al., 2008). For the iterative methods, the smallest n is determined whose interval width does not exceed $2 \cdot \text{MoE}$.

The selected focus scenario is marked by an arrow ($\leftarrow$) in the table.

Table 1: Comparison of the required net sample sizes by method

CI (%)	MoE ($\pm$ pp)	Wald / Cochran	Wilson score	Clopper-Pearson	Bayes (credible)
90	1	118	118	118	118
90	2	113	113	113	113
90	5	84	84	85	84
90	10	44	43	47	43
95	1	119	119	119	119
95	2	115	115	115	115
95 $\leftarrow$	5	92	92	93	92
95	10	54	53	56	53
99	1	120	120	120	120
99	2	117	117	117	117
99	5	102	102	103	102
99	10	70	69	71	70

4 Efficiency Analysis of Cluster Sampling

Animals in housing systems are often not completely independent observations. Animals in the same pen share air, feed and management, so that diseases or behaviours become more similar. The **intra-class correlation coefficient** (ICC , ρ) measures this effect ([Dohoo et al., 2003](#)).

Status: Cluster planning deactivated.

The calculation assumes true simple random sampling across the entire herd, without pen effects. The design effect is 1.0. The following sample sizes result for the focus scenario:

Table 2: Sample sizes for the focus scenario (simple random sampling)

Statistical method	Animals to be examined (n)
Wald / Cochran	92
Wilson score	92
Clopper-Pearson	93
Bayes (credible)	92

5 References

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